**Three Winding Transformer Test System**

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February 2014

Revised by R. Dugan and D. Montenegro, Rev 3

August 6, 2016

Revised by E. Schweitzer, Rev 4

July 26, 2017

# Introduction

The system is to be used for testing a common three winding transformer configuration found in distribution systems and shown in Figure 1.



Figure 1 – Test System

# Test Case Data

***Transformer Ratings:***



***Per-Unit Short-Circuit (Leakage) impedances based upon 15,000 kVA and rated line-to-line voltages***

******pu@15,000 kVA

***Transmission line 1-2:***



***Distribution line 3-4:***



***Distribution line 5-6:***



***Bus 4 Constant PQ Load:***

 kW+jkvar

***Bus 6 Constant Z Load:***

 kW+jkvar

***Infinite Bus:*** Balanced line-to-line 69 kV

## Three-phase transformer circuit:

OpenDSSTransConnections

Figure 2 – Transformer circuit

The Test System was built in the OpenDSS program. The transformer is modeled using three 1-phase transformers, each with 3 windings. They are connected as shown in the Figure 2. OpenDSS can also model the transformer as a 3-phase transformer, but the delta winding would be “buried” and the delta winding currents are not available. By constructing the model with three 1-phase transformers, it is possible for OpenDSS to report the internal delta currents as well as the line currents in the line from Bus 5 to Bus 6.

# Results

The following results are obtained in OpenDSS. Results were additionally confirmed in Mathcad and CYMDIST

## Voltages

Line-Ground and Line-Line Voltages by Bus and Node

Bus Node VLN (kV) Angle pu Base kV Node-Node VLL (kV) Angle pu

B1 . 1 39.836 /\_ 0.0 0.99997 69.000 1-2 68.997 /\_ 30.0 0.99996

- 2 39.836 /\_ -120.0 0.99996 69.000 2-3 68.997 /\_ -90.0 0.99996

- 3 39.836 /\_ 120.0 0.99997 69.000 3-1 68.998 /\_ 150.0 0.99997

B2 . 1 39.464 /\_ -0.3 0.99063 69.000 1-2 68.401 /\_ 29.6 0.99132

- 2 39.443 /\_ -120.5 0.99011 69.000 2-3 68.428 /\_ -90.4 0.99171

- 3 39.615 /\_ 119.6 0.99442 69.000 3-1 68.457 /\_ 149.6 0.99213

B3 . 1 7.44 /\_ -2.1 0.97625 13.200 1-2 12.871 /\_ 27.6 0.97508

- 2 7.4008 /\_ -122.4 0.9711 13.200 2-3 12.86 /\_ -92.0 0.97424

- 3 7.4861 /\_ 118.1 0.9823 13.200 3-1 12.94 /\_ 147.9 0.98033

B4 . 1 7.3091 /\_ -2.3 0.95908 13.200 1-2 12.695 /\_ 27.1 0.96171

- 2 7.2743 /\_ -123.4 0.9545 13.200 2-3 12.712 /\_ -92.5 0.96302

- 3 7.4646 /\_ 117.4 0.97948 13.200 3-1 12.781 /\_ 147.2 0.96825

B5 . 1 1.3283 /\_ -37.9 0.9586 2.400 1-2 2.2982 /\_ -4.7 0.95759

- 2 1.3912 /\_ -153.2 1.004 2.400 2-3 2.2832 /\_ -124.6 0.95135

- 3 1.2546 /\_ 87.6 0.90541 2.400 3-1 2.2959 /\_ 115.7 0.95661

B6 . 1 1.3181 /\_ -38.2 0.95123 2.400 1-2 2.2826 /\_ -5.0 0.95109

- 2 1.3832 /\_ -153.5 0.99821 2.400 2-3 2.2694 /\_ -124.9 0.94559

- 3 1.2458 /\_ 87.2 0.89905 2.400 3-1 2.2784 /\_ 115.4 0.94935

## Powers

Lines

ELEMENT = "Line.TLINE1-2"

kW kvar kVA PF

B1 1 2016.8 +j 851.9 2189.4 0.9212

B1 2 2134.9 +j 1167.3 2433.2 0.8774

B1 3 1699.0 +j 705.7 1839.7 0.9235

TERMINAL TOTAL 5850.6 +j 2725.0 6454.1 0.9065

ELEMENT = "Line.DLINE3-4"

B3 1 1270.4 +j 513.2 1370.1 0.9272

B3 2 1513.2 +j 787.9 1706.0 0.8870

B3 3 1000.1 +j 261.5 1033.7 0.9675

TERMINAL TOTAL 3783.6 +j 1562.6 4093.6 0.9243

ELEMENT = "Line.DLINE5-6"

B5 1 701.8 +j 304.5 765.0 0.9174

B5 2 702.3 +j 379.1 798.1 0.8800

B5 3 629.4 +j 346.4 718.4 0.8761

TERMINAL TOTAL 2033.6 +j 1030.0 2279.5 0.8921

Transformer

ELEMENT = "Transformer.YYDA"

B2 1 2010.2 +j 923.8 2212.3 0.9086

TERMINAL TOTAL 2010.2 +j 923.8 2212.3 0.9086

B3 1 -1270.4 +j -513.2 1370.1 0.9272

TERMINAL TOTAL -1270.4 +j -513.2 1370.1 0.9272

B5 1 -450.4 +j 90.4 459.4 -0.9804

B5 2 -287.5 +j -385.8 481.2 0.5976

TERMINAL TOTAL -737.9 +j -295.4 794.9 0.9284

ELEMENT = "Transformer.YYDB"

B2 2 2118.1 +j 1226.8 2447.8 0.8653

TERMINAL TOTAL 2118.1 +j 1226.8 2447.8 0.8653

B3 2 -1513.2 +j -787.9 1706.0 0.8870

TERMINAL TOTAL -1513.2 +j -787.9 1706.0 0.8870

B5 2 -414.8 +j 6.7 414.9 -0.9999

B5 3 -187.8 +j -323.6 374.1 0.5021

TERMINAL TOTAL -602.6 +j -316.9 680.9 0.8851

ELEMENT = "Transformer.YYDC"

B2 3 1694.5 +j 773.5 1862.7 0.9097

TERMINAL TOTAL 1694.5 +j 773.5 1862.7 0.9097

B3 3 -1000.1 +j -261.5 1033.7 0.9675

TERMINAL TOTAL -1000.1 +j -261.5 1033.7 0.9675

B5 3 -441.6 +j -22.8 442.2 0.9987

B5 1 -251.4 +j -394.9 468.1 0.5371

TERMINAL TOTAL -693.0 +j -417.7 809.2 0.8565

Loads

Bus Phase kW +j kvar kVA PF

ELEMENT = "Load.B4A"

B4 1 1250.0 +j 500.0 1346.3 0.9285

B4 0 0.0 +j 0.0 0.0 1.0000

TERMINAL TOTAL 1250.0 +j 500.0 1346.3 0.9285

ELEMENT = "Load.B4B"

B4 2 1500.0 +j 750.0 1677.0 0.8944

B4 0 0.0 +j 0.0 0.0 1.0000

TERMINAL TOTAL 1500.0 +j 750.0 1677.0 0.8944

ELEMENT = "Load.B4C"

B4 3 1000.0 +j 250.0 1030.8 0.9701

B4 0 0.0 +j 0.0 0.0 1.0000

TERMINAL TOTAL 1000.0 +j 250.0 1030.8 0.9701

ELEMENT = "Load.B6"

B6 1 697.9 +j 298.6 759.2 0.9194

B6 2 700.1 +j 373.6 793.5 0.8822

B6 3 627.0 +j 340.3 713.4 0.8789

TERMINAL TOTAL 2025.0 +j 1012.5 2264.0 0.894

## Currents

Line 1-2

ELEMENT = "Line.TLINE1-2"

|I| Angle re im

B1 1 54.959 /\_ -22.9 = 50.627 +j -21.388

B1 2 61.08 /\_ -148.7 = -52.175 +j -31.758

B1 3 46.183 /\_ 97.4 = -5.9806 +j 45.794

B1 Resid 10.523 /\_ 44.3 = 7.529 +j 7.3524

Line 3-4

ELEMENT = "Line.DLINE3-4"

B3 1 184.16 /\_ -24.1 = 168.07 +j -75.271

B3 2 230.52 /\_ -149.9 = -199.47 +j -115.55

B3 3 138.08 /\_ 103.4 = -32.072 +j 134.3

B3 Resid 84.983 /\_ 41.7 = 63.47 +j 56.512

Line 5-6

ELEMENT = "Line.DLINE5-6"

B5 1 575.96 /\_ -61.3 = 276.26 +j -505.38

B5 2 573.69 /\_ 178.4 = -573.47 +j 15.912

B5 3 572.64 /\_ 58.7 = 297.21 +j 489.47

Transformer – Phase A

B2 1 56.06 /\_ -25.0 = 50.799 +j -23.71

B3 1 184.16 /\_ 155.9 = -168.07 +j 75.271

------Delta winding------

B5 1 345.86 /\_ 153.5 = -309.43 +j 154.5

Transformer – Phase B

B2 2 62.059 /\_ -150.6 = -54.071 +j -30.456

B3 2 230.52 /\_ 30.1 = 199.47 +j 115.55

------Delta winding------

B5 2 298.2 /\_ 27.7 = 264.04 +j 138.59

Transformer – Phase C

B2 3 47.019 /\_ 95.0 = -4.1375 +j 46.837

B3 3 138.08 /\_ -76.6 = 32.072 +j -134.3

------Delta winding------

B5 3 352.44 /\_ -95.4 = -33.165 +j -350.88

# OpenDSS Transformer Model

The OpenDSS program computes the primitive Y matrix for each transformer directly from the per unit short circuit impedances between each pair of windings. The primitive Y matrix is then used to help build the system Y matrix (nodal admittance matrix). An equivalent circuit model is not created.

The procedure is to build a short-circuit Z matrix similar to what is done to construct the short-circuit matrix for a transmission network using the infinite bus as the reference. This requires *m(m-1)/2* short circuit measurements where *m* is the number of ports of the network. Fortunately, this is what is available from the typical transformer test report or from the nameplate. Also, fortunately, these values can be obtained with sufficient accuracy to build the transformer leakage impedance model. One does not have to build the tightly coupled L matrix that can lead to errors.

Using the first winding as the reference winding, the short-circuit matrix, **Z**B,  is constructed as follows:

Diagonal Elements of **Z**B

zBii = zSC *1*, *i+1* × zbase for *i* = 1 to *m-1* (1)

where *m*= number of windings. zSC1,i  = short circuit impedance from winding 1 to winding *i* in per unit on some base. The short circuit impedance measurements are always between pairs of windings: there is no such thing as the short circuit impedance of one winding.

Off-diagonal Elements of **Z**B:

zBij = 0.5[zBii + zBjj - zScj+1,*i+1* × zbase] *i≠ j* (2)

Note that the order of **Z**B is *m-1* where *m* is the number of windings.

After *ZB* is constructed, it is inverted and transformed through a series of power invariant reference frame transformations

**Y**prim = **A N B Z**B-1 **B**T **N**T **A**T (3)

**Y**w

**Y**1

For each single-phase transformer in the bank of transformers making up the 3-phase, three winding unit, the values of these matrices is as follows. The B transformation matrices convert from the short-circuit reference frame to Y1 on a one-voltage, or per unit, basis. The N matrices convert Y1 to actual S given the turns ratios or voltage ratings. (OpenDSS generally uses the voltage ratings because Y1 is built on a 1-voltage base.) Finally, the A transformation takes into account the actual terminal connections of the transformer winding and we end up with Y in actual values ready to be added into the system Y matrix.

As a result, the transformer model will be connected to each bus as shown in Figure 3. Notice that each bus connection varies according to the winding connection mode (wye/delta), this can be seen when declaring the transformers on the OpenDSS script at the end of the document. Figure 4 shows the single line diagram for the proposed circuit.

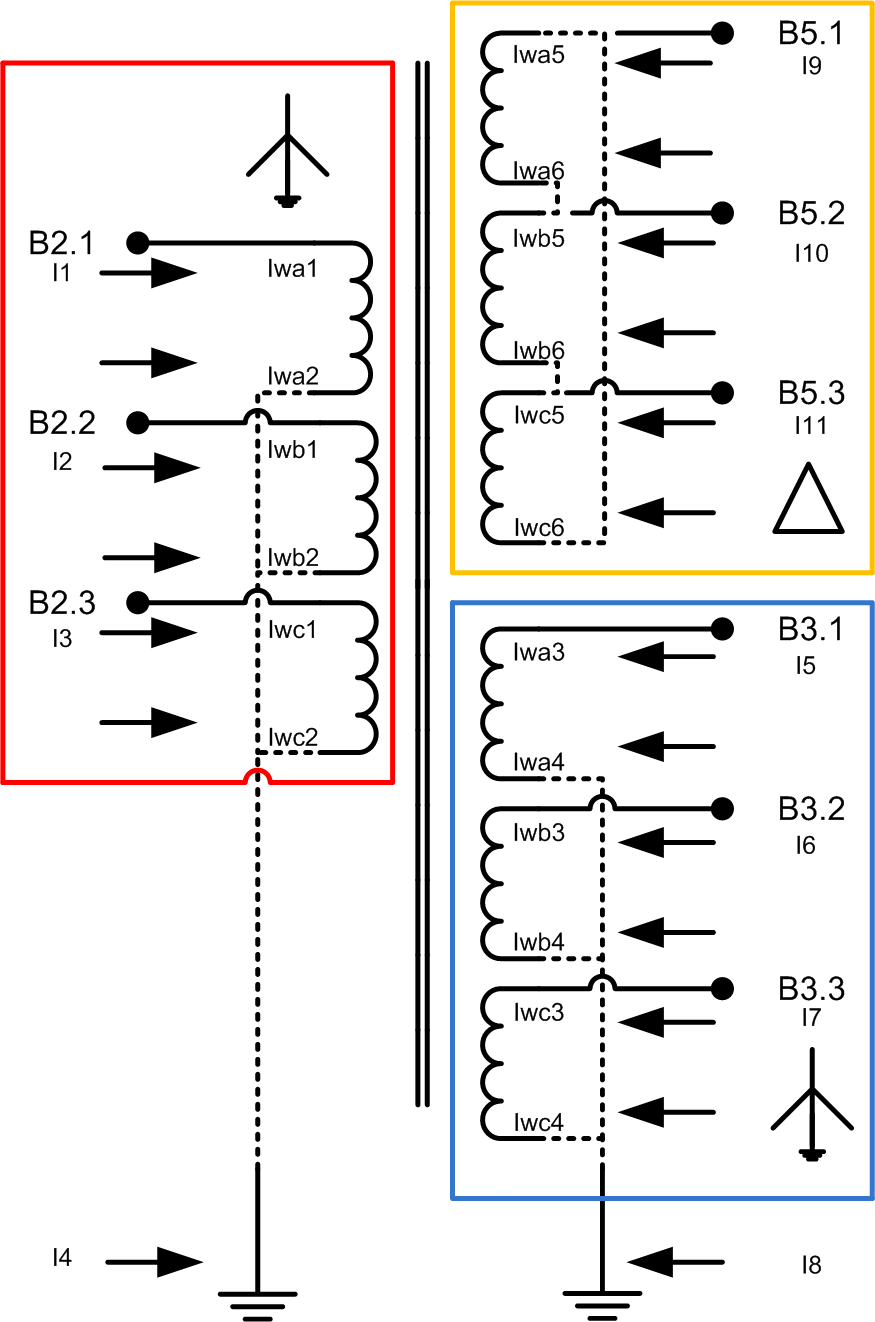


Figure 3. YYD transformer connected to Buses B2, B3 and B5.

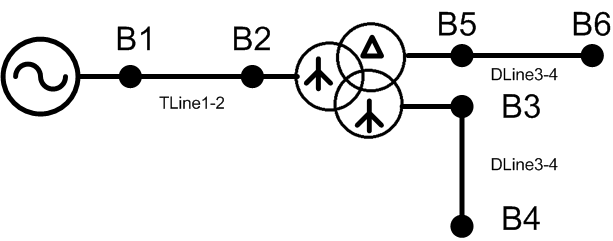


Figure 4. Circuit scripted using OpenDSS for testing the YYD transformer.

According to Figure 3, the matrix *A* will describe the relationships between the Y primitive matrix and its connection buses as follows:

As can be seen in (2), the number after the dot (.) of each bus indicates the connection point/node where the conductor of the transformer is connected.

Here are the values dumped from OpenDSS for one of the 1-phase transformer units:

ZB: (R matrix first, then X matix in lower triangle form)

4.4E-010

2.2E-010 7.7E-010

1.5E-008

1.55E-008 7.2E-008

ZB: (inverted)

2797571.4578

-537767.9179 250401.1922

-85648124.6069

18440934.9421 -17857777.6615

Y\_OneVolt

1972436.8142

-2259803.5399 2797571.4578

287366.7257 -537767.9179 250401.1922

-66624032.3842

67207189.6648 -85648124.6069

-583157.2806 18440934.9421 -17857777.6615

Y\_Terminal (Siemens)

0.0012

-0.0012 0.0012

-0.0074 0.0074 0.0482

0.0074 -0.0074 -0.0482 0.0482

0.0030 -0.0030 -0.0294 0.0294 0.0435

-0.0030 0.0030 0.0294 -0.0294 -0.0435 0.0435

-0.0420

0.0420 -0.0420

0.2214 -0.2214 -1.4751

-0.2214 0.2214 1.4751 -1.4751

-0.0061 0.0061 1.0084 -1.0084 -3.1003

0.0061 -0.0061 -1.0084 1.0084 3.1003 -3.1003

# Single-Line-to-Ground (SLG) Fault Case

There are two cases of interest for a SLG fault:

1. SLG fault at Bus 4
2. SLG fault at Bus 2

Case 1 is the fault of most interest for Distribution System Analysis. However, the delta winding in the transformer will also cause the transformer to participate in SLG faults on the transmission system (HV) side of the substation. Both cases are presented here.

In these cases, loads will be ignored and the currents computed solely using line and transformer impedances.

## Case 1: SLG Fault at Bus 4

### Voltages

LINE-GROUND and LINE-LINE VOLTAGES BY BUS & NODE

Bus Node VLN (kV) Angle pu Base kV Node-Node VLL (kV) Angle pu

B1 . 1 39.816 /\_ 0.0 0.99947 69.000 1-2 68.977 /\_ 30.0 0.99967

- 2 39.835 /\_ -120.0 0.99994 69.000 2-3 69 /\_ -90.0 1

- 3 39.835 /\_ 120.0 0.99995 69.000 3-1 68.978 /\_ 150.0 0.99968

B2 . 1 33.489 /\_ 0.9 0.84064 69.000 1-2 65.292 /\_ 32.7 0.94626

- 2 40.83 /\_ -121.7 1.0249 69.000 2-3 69.147 /\_ -90.4 1.0021

- 3 40.313 /\_ 121.4 1.0119 69.000 3-1 64.158 /\_ 148.1 0.92982

B3 . 1 3.9141 /\_ -8.1 0.5136 13.200 1-2 9.7468 /\_ 38.3 0.7384

- 2 7.5961 /\_ -119.8 0.99672 13.200 2-3 13.229 /\_ -90.4 1.0022

- 3 7.5881 /\_ 119.0 0.99568 13.200 3-1 10.428 /\_ 136.5 0.78997

**B4 . 1 0.000338 /\_ -74.5 0.00004 13.200 1-2 8.7268 /\_ 49.0 0.66112**

**- 2 8.727 /\_ -131.0 1.1451 13.200 2-3 13.379 /\_ -92.6 1.0135**

**- 3 8.4982 /\_ 127.0 1.1151 13.200 3-1 8.4985 /\_ 127.0 0.64382**

B5 . 1 1.1216 /\_ -47.7 0.80947 2.400 1-2 1.5845 /\_ -4.8 0.66019

- 2 1.0776 /\_ -139.8 0.77766 2.400 2-3 2.1704 /\_ -111.7 0.90433

- 3 1.3213 /\_ 90.9 0.95354 2.400 3-1 2.2858 /\_ 109.8 0.9524

B6 . 1 1.1216 /\_ -47.7 0.80947 2.400 1-2 1.5845 /\_ -4.8 0.66019

- 2 1.0776 /\_ -139.8 0.77766 2.400 2-3 2.1704 /\_ -111.7 0.90433

- 3 1.3213 /\_ 90.9 0.95354 2.400 3-1 2.2858 /\_ 109.8 0.9524

### Currents

|I| Angle re im

ELEMENT = "Line.TLINE1-2"

B1 1 574.1 /\_ -73.7 = 160.7 +j -551.15

B1 2 70.464 /\_ 98.1 = -9.9811 +j 69.754

B1 3 71.339 /\_ 101.1 = -13.76 +j 69.999

B1 Resid 433.59 /\_ 108.4 = -136.96 +j 411.4

ELEMENT = "Line.DLINE3-4"

B3 1 3386.4 /\_ -74.5 = 903.27 +j -3263.7

B3 2 0.057348 /\_ -31.7 = 0.048781 +j -0.030153

B3 3 0.048308 /\_ -155.4 = -0.043928 +j -0.0201

B3 Resid 3386.4 /\_ 105.5 = -903.28 +j 3263.8

ELEMENT = "Line.DLINE5-6"

B5 1 0.00020849 /\_ 53.4 = 0.00012424 +j 0.00016742

B5 2 0.00021448 /\_ -56.2 = 0.00011928 +j -0.00017825

B5 3 0.00024377 /\_ 177.5 = -0.00024353 +j 1.0827E-005

B5 Resid 5.457E-012 /\_ 90.0 = 0 +j 5.457E-012

ELEMENT = "Transformer.YYDA"

B2 1 576.25 /\_ -73.8 = 160.9 +j -553.33

B3 1 3386.4 /\_ 105.5 = -903.27 +j 3263.7

------Delta winding------

B5 1 1195.7 /\_ -80.5 = 197.65 +j -1179.2

ELEMENT = "Transformer.YYDB"

B2 2 72.04 /\_ 99.5 = -11.9 +j 71.05

B3 2 0.057348 /\_ 148.3 = -0.048781 +j 0.030153

------Delta winding------

B5 2 1195.7 /\_ -80.5 = 197.65 +j -1179.2

ELEMENT = "Transformer.YYDC"

B2 3 72.045 /\_ 99.5 = -11.918 +j 71.052

B3 3 0.048308 /\_ 24.6 = 0.043928 +j 0.0201

------Delta winding------

B5 3 1195.7 /\_ -80.5 = 197.65 +j -1179.2

**Fault Current**

ELEMENT = "Fault.SLGFAULT"

B4 1 3386.4 /\_ -74.5 = 903.28 +j -3263.7

## Case 2: SLG Fault at Bus 2 (Transmission side)

### Voltages

LINE-GROUND and LINE-LINE VOLTAGES BY BUS & NODE

Bus Node VLN (kV) Angle pu Base kV Node-Node VLL (kV) Angle pu

B1 . 1 39.713 /\_ 0.0 0.99689 69.000 1-2 68.87 /\_ 30.0 0.99811

- 2 39.826 /\_ -120.0 0.99972 69.000 2-3 69 /\_ -90.0 1

- 3 39.831 /\_ 120.0 0.99984 69.000 3-1 68.885 /\_ 149.9 0.99833

**B2 . 1 0.00035 /\_ -70.1 8.9E-6 69.000 1-2 49.397 /\_ 48.1 0.71589**

**- 2 49.397 /\_ -131.9 1.24 69.000 2-3 70.047 /\_ -92.1 1.0152**

**- 3 45.004 /\_ 132.4 1.1297 69.000 3-1 45.004 /\_ 132.4 0.65223**

B3 . 1 0.87097 /\_ 3.0 0.11429 13.200 1-2 9.4503 /\_ 48.1 0.71593

- 2 8.8571 /\_ -127.9 1.1622 13.200 2-3 13.401 /\_ -92.1 1.0152

- 3 8.085 /\_ 127.7 1.0609 13.200 3-1 8.6099 /\_ 132.4 0.65227

B4 . 1 0.87097 /\_ 3.0 0.11429 13.200 1-2 9.4503 /\_ 48.1 0.71593

- 2 8.8571 /\_ -127.9 1.1622 13.200 2-3 13.401 /\_ -92.1 1.0152

- 3 8.085 /\_ 127.7 1.0609 13.200 3-1 8.6099 /\_ 132.4 0.65227

B5 . 1 0.9423 /\_ -51.7 0.68004 2.400 1-2 1.2742 /\_ 3.2 0.5309

- 2 1.0637 /\_ -130.3 0.76765 2.400 2-3 2.2606 /\_ -108.4 0.94193

- 3 1.3339 /\_ 88.9 0.96265 2.400 3-1 2.1468 /\_ 105.1 0.89449

B6 . 1 0.9423 /\_ -51.7 0.68004 2.400 1-2 1.2742 /\_ 3.2 0.5309

- 2 1.0637 /\_ -130.3 0.76765 2.400 2-3 2.2606 /\_ -108.4 0.94193

- 3 1.3339 /\_ 88.9 0.96265 2.400 3-1 2.1468 /\_ 105.1 0.89449

### Currents

ELEMENT = "Line.TLINE1-2"

B1 1 3403.3 /\_ -69.2 = 1208.8 +j -3181.4

B1 2 183.64 /\_ 93.3 = -10.438 +j 183.34

B1 3 184.12 /\_ 94.4 = -14.24 +j 183.57

B1 Resid 3053.4 /\_ 112.8 = -1184.1 +j 2814.5

ELEMENT = "Line.DLINE3-4"

B3 1 0.025574 /\_ 119.3 = -0.012499 +j 0.022311

B3 2 0.060538 /\_ -31.2 = 0.051797 +j -0.031336

B3 3 0.047576 /\_ -153.2 = -0.042457 +j -0.021467

B3 Resid 0.030655 /\_ 84.1 = 0.0031602 +j 0.030492

ELEMENT = "Line.DLINE5-6"

B5 1 0.00016736 /\_ 51.7 = 0.0001037 +j 0.00013136

B5 2 0.00020649 /\_ -45.8 = 0.00014404 +j -0.00014796

B5 3 0.0002483 /\_ 176.2 = -0.00024774 +j 1.6599E-005

B5 Resid 2.4824E-011 /\_ 28.4 = 2.1828E-011 +j 1.1823E-011

ELEMENT = "Transformer.YYDA"

B2 1 185.12 /\_ 93.8 = -12.412 +j 184.7

B3 1 0.025574 /\_ -60.7 = 0.012499 +j -0.022311

------Delta winding------

B5 1 3072.1 /\_ -86.2 = 205.95 +j -3065.2

ELEMENT = "Transformer.YYDB"

B2 2 185.11 /\_ 93.8 = -12.4 +j 184.69

B3 2 0.060538 /\_ 148.8 = -0.051797 +j 0.031336

------Delta winding------

B5 2 3072.1 /\_ -86.2 = 205.95 +j -3065.2

ELEMENT = "Transformer.YYDC"

B2 3 185.11 /\_ 93.8 = -12.418 +j 184.69

B3 3 0.047576 /\_ 26.8 = 0.042457 +j 0.021467

------Delta winding------

B5 3 3072.1 /\_ -86.2 = 205.95 +j -3065.2

**Fault current**

ELEMENT = "Fault.SLGFAULT"

B2 1 3582.2 /\_ -70.1 = 1221.4 +j -3367.5